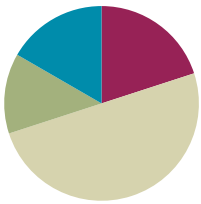
Mathematics Level C: Lesson 1 – Equivalent Fractions

Targeted Standards:

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Lesson	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Sprint: Equivalent Fractions Fluency Practice

[Note: A "Sprint" is a timed practice of a specific skill, usually on a single worksheet that is repeated so that students can improve their time and accuracy.]

Equivalent Fractions (3 minutes)

T: (Write.) $\frac{1}{2} =$

T: Say the fraction.

S: One half.

T: (Write.) $\frac{1}{2} = \frac{\quad}{4}$

T: One half is how many fourths?

S: Two fourths.

Continue with possible sequence:

$\frac{1}{2} = \frac{\quad}{6}, \frac{1}{3} = \frac{\quad}{6}, \frac{2}{3} = \frac{\quad}{6}, \frac{2}{3} = \frac{\quad}{12}, \frac{3}{4} = \frac{\quad}{16}, \frac{3}{5} = \frac{\quad}{25}$

T: (Write.) $\frac{1}{2} =$

T: Say the fraction.

S: One half.

T: (Write.) $\frac{1}{2} = \frac{2}{\quad}$

T: One half or one part of two is the same as two parts of what unit?

S: Fourths.

Repeat with another possible sequence:

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Equivalent Fractions is intentionally placed before the Sprint because it reviews the Sprint skill. Meet the needs of your students by adjusting the amount of time you spend on it. If you find that students struggle to complete Sprint A, you may want to do another minute or two of Equivalent Fractions before moving them on to Sprint B.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

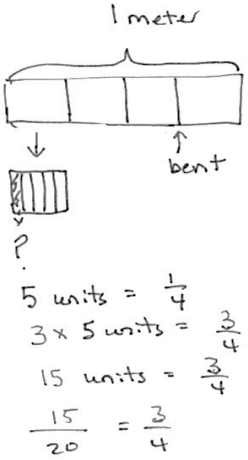
Adjusting number words and correctly pronouncing them as fractions (fifths, sixths, etc.) may be challenging. If you have many ELLs, before starting you might quickly count together to practice enunciating word endings: hal**ves**, thir**ds**, four**ths**, fif**ths**, six**ths**, etc.

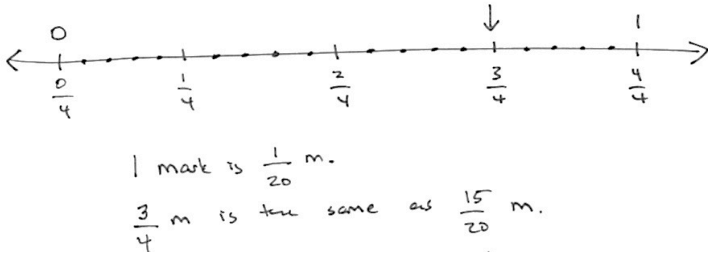
$\frac{1}{2}=\frac{2}{\_},\frac{1}{5}=\frac{2}{\_},\frac{2}{5}=\frac{8}{\_},\frac{2}{5}=\frac{8}{\_},\frac{3}{4}=\frac{9}{\_},\frac{4}{5}=\frac{16}{\_}$

Application Problem (8 minutes)

Mr. Hopkins has a 1-meter wire he is using to make clocks. Each fourth meter is marked off with 5 smaller equal lengths. If Mr. Hopkins bends the wire at ¾ meter, what fraction of the marks is that?

T: (After the students have solved the problem, possibly using the RDW process independently or in partners.) Let's look at two of your solutions and compare them.





NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students have been using the RDW process since Level 1: Read, Draw, Write.

1. Read the problem.
2. Draw to represent the problem*.
3. Write an equation(s) that either helps you to solve the problem or shows how you solved it.
4. Write a statement of the answer to the question.

*Embedded within D are important reflective questions:

- What do I see?
- Can I draw something?
- What conclusions can I make from my drawing?

T: When you look at these two solutions side by side what do you see? (You might use the following set of questions to help students compare the solutions as a whole class, or to encourage inter-partner communication as you circulate while they compare.)

- What did each of these students draw?
- What conclusions can you make from their drawings?
- How did they record their solutions numerically?
- How does the tape diagram relate to the number line?
- What does the tape diagram/number line clarify?
- What does the equation clarify?
- How could the statement with the number line be rephrased to answer the question?

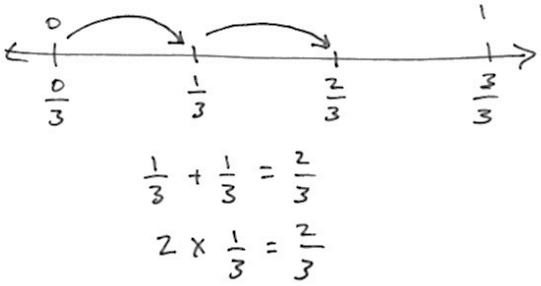
Concept Development (30 minutes)

Materials: (S) Blank paper

Problem 1

$$\frac{1}{3} + \frac{1}{3}$$

1 third + 1 third = 2 thirds

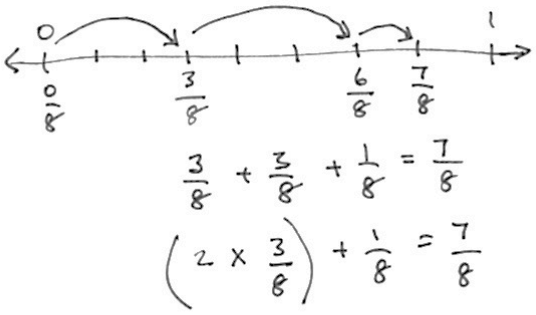


T: On a number line, mark the end points as zero and 1. Between zero and 1 estimate to make three parts of equal length and label them with their fractional value.
T: (After students work.) On your number line, show 1 third plus 1 third with arrows designating lengths. (Demonstrate and then pause as students work).
T: The answer is?
S: 2 thirds.
T: Talk to your partner. Express this as a multiplication equation and as an addition sentence.
S: $\frac{1}{3}+\frac{1}{3}=2\times\frac{1}{3}=\frac{2}{3}$
T: Following the same pattern of adding unit fractions by joining lengths, show 3 fourths on a number line.

Problem 2

$$\frac{3}{8}+\frac{3}{8}+\frac{1}{8}$$

3 eighths + 3 eighths + 1 eighth



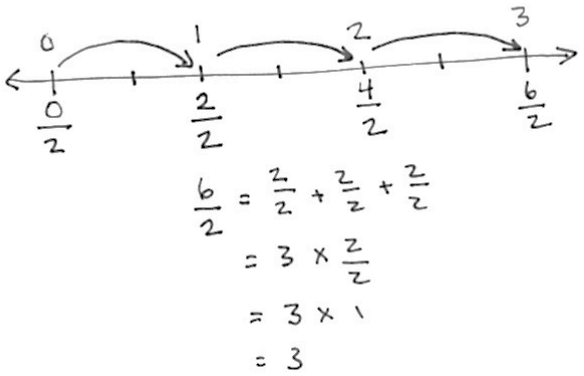
T: On a number line, again mark the end points as zero and one. Between zero and one, estimate to make 8 parts of equal length. This time only label what is necessary to show 3 eighths.
T: (After students work.) Represent 3 eighths + 3 eighths + 1 eighth on your number line.
T: The answer is?
S: 7 eighths.
T: Talk to your partner. Express this as a multiplication equation and as an addition equation.
S: $\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=\left(2\times\frac{3}{8}\right)+\frac{1}{8}=\frac{7}{8}$

Problem 3

$$\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3$$

6 halves = 3 x 2 halves = 3 ones = 3

T: On a number line, mark the end points as 0 halves and 6 halves below the number line. Estimate to make 6 parts of equal length. This time only label 2 halves.
T: (After students work.) Record the whole number equivalents above the line.
T: Represent 3 x 2 halves on your number line.
T: (After students have worked) The answer is?
S: 6 halves or 3.
T: 3. What is the unit?
S: 3 ones.
T: Talk to your partner. Express this as an addition equation and as an multiplication equation.
S: $\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 3 \times \frac{2}{2} = 3$

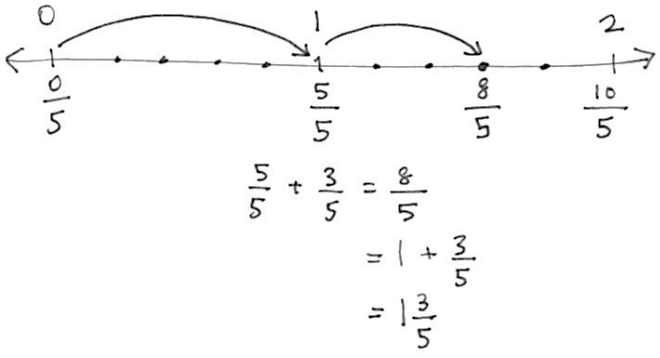


Problem 4

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$

8 fifths = 5 fifths + 3 fifths = 1 and 3 fifths

T: Use a number line. Mark the end points as 0 fifths and 10 fifths below it. Estimate and give a value to the halfway point.
T: What will be the value of the halfway point?
S: 5 fifths.
T: Make 10 parts of equal length from 0 fifths to 10 fifths.
T: (After students work.) Record the whole number equivalents above the line.
T: (After students work.) Label 8 fifths on your number line.
T: Show 8 fifths as the sum of 5 fifths and 3 fifths on your number line.
S: (After students work.)
T: Talk to your partner. Express this as an addition equation in two ways: as the sum of fifths and as the sum of a whole number and fifths.
T: (After students work.) Another way of expressing 1 plus 3 fifths is?
S: 1 and 3 fifths.
S: $\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 3 \times \frac{2}{2} = 3$
T: 8 fifths is between what 2 whole numbers?
S: 1 and 2.

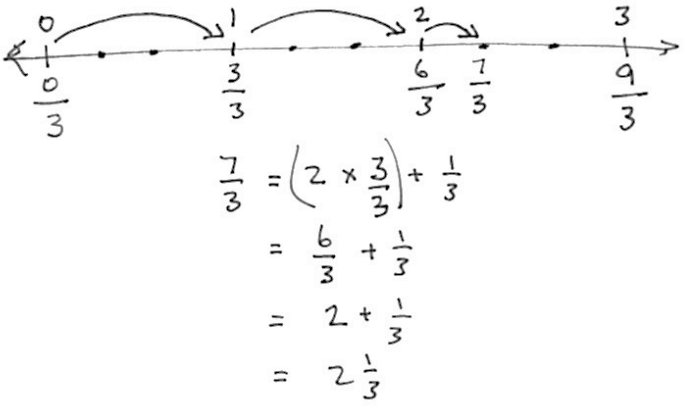


Problem 5

$$\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

7 thirds = 6 thirds + 1 third = 2 and 1 third.

T: Use a number line. Mark the end points as 0 thirds and 9 thirds below the number line. Divide the whole length into three equal smaller lengths and mark their values using thirds. Work with a partner.
T: (After students work). What are the values of those points?
S: 3 thirds and 6 thirds.
T: Mark the whole number equivalents above the line.
T: (After students work.) Divide each of those whole number lengths into three smaller lengths. Mark the number 7 thirds.
T: (After students work.) Show 7 thirds as two units of 3 thirds and one more third on your number line and in an equation. Work together if you get stuck.
T: (After working and dialogue) 7 thirds is between what two whole numbers?
S: 2 and 3.



Student Debrief (10 minutes)

Lesson Objective: Making equivalent fractions with sums of fractions with like denominators.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

T: Come to the Debrief and bring your Problem Set. Compare your work to your neighbor's. On which problems do you have different answers? Discuss your differences. Both may be correct.
T: (After about 3 minutes.)
T: What is a way to express 3/7 as a sum?
S: 1 sevenths + 1 seventh + 1 seventh.
T: Another way?
S: 2 sevenths + 1 seventh.
T: These are equivalent forms of 3 sevenths.
T: On your Problem Set find and talk to your partner about different equivalent forms of your numbers.
S: 6 sevenths could be expressed as 3 sevenths + 3 sevenths or as 3 times 2 sevenths. → 9 sevenths can be expressed as 1 + 2 sevenths. → 7 fourths can be expressed as 2 times 3 fourths + 1 fourth. → 1 and 3 fourths can be expressed as 7 fourths. → 32 sevenths can be expressed as 28 sevenths + 4 sevenths or 4 and 4 sevenths.

T: I'm hearing you express these numbers in many equivalent forms. Why do you think I chose to use the tool of the number line in this lesson? Talk this over with your partner. If you were the teacher of this lesson, why might you use the number line?
T: (After students discuss.) When we were studying decimal place value, we saw that 9 tenths + 3 tenths is equal to 12 tenths or 1 + 2 tenths or 1 and 2 tenths.
T: Once more, please review the solution and number line you made for question 4 about Marisela's ribbon. Discuss the equivalence of 20 eighths and 2 and 4 eighths as it relates to the number line.
T: (After students talk.) Discuss the relationship of the equivalence of these sums.

9 tenths + 3 tenths = 12 tenths = 1 + 2 tenths = $1\frac{1}{10}$.

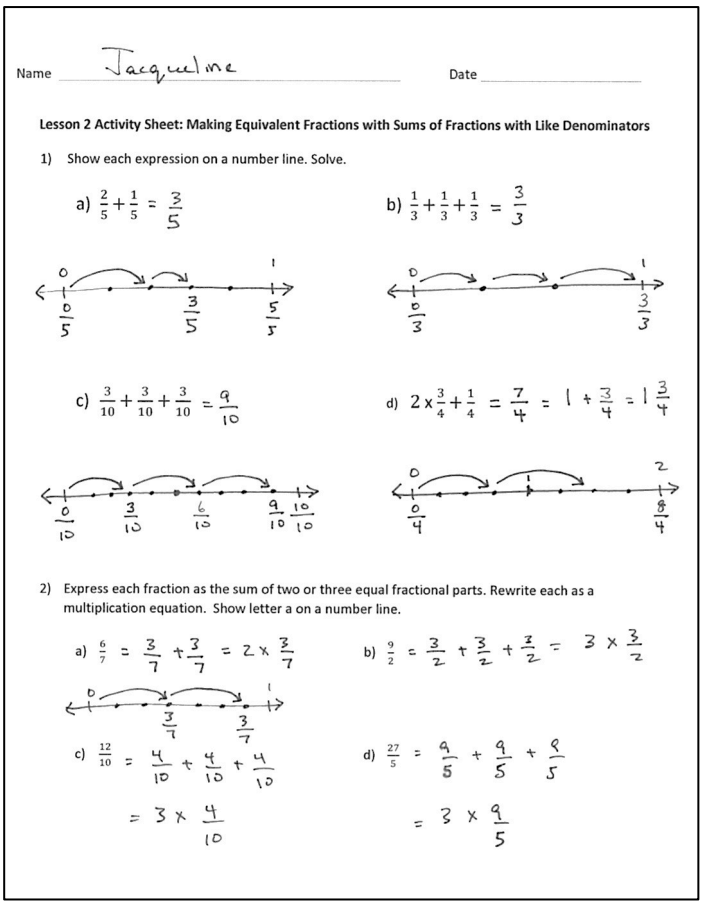
9 elevenths + 3 elevenths = 12 elevenths = 1 + 1 eleventh = $1\frac{1}{11}$.

T: (After students talk.) Yes, our place value system is another example of equivalence.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Sample:



Name Jacqueline Date

Lesson 2 Activity Sheet: Making Equivalent Fractions with Sums of Fractions with Like Denominators

1) Show each expression on a number line. Solve.

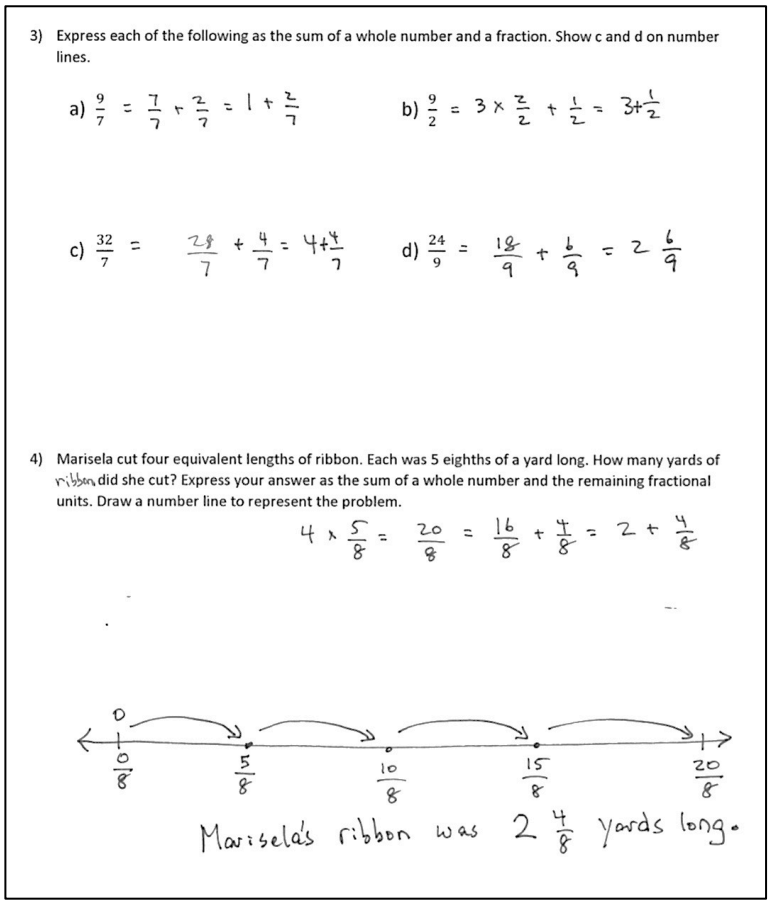
a) $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$ b) $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}$

c) $\frac{3}{10}+\frac{3}{10}+\frac{3}{10}=\frac{9}{10}$ d) $2\times\frac{3}{4}+\frac{1}{4}=\frac{7}{4}=1+\frac{3}{4}=1\frac{3}{4}$

2) Express each fraction as the sum of two or three equal fractional parts. Rewrite each as a multiplication equation. Show letter a on a number line.

a) $\frac{6}{7}=\frac{3}{7}+\frac{3}{7}=2\times\frac{3}{7}$ b) $\frac{9}{2}=\frac{3}{2}+\frac{3}{2}+\frac{3}{2}=3\times\frac{3}{2}$

c) $\frac{12}{10}=\frac{4}{10}+\frac{4}{10}+\frac{4}{10}=3\times\frac{4}{10}$ d) $\frac{27}{5}=\frac{9}{5}+\frac{9}{5}+\frac{9}{5}=3\times\frac{9}{5}$



3) Express each of the following as the sum of a whole number and a fraction. Show c and d on number lines.

a) $\frac{9}{7}=\frac{7}{7}+\frac{2}{7}=1+\frac{2}{7}$ b) $\frac{9}{2}=3\times\frac{2}{2}+\frac{1}{2}=3+\frac{1}{2}$

c) $\frac{32}{7}=\frac{28}{7}+\frac{4}{7}=4+\frac{4}{7}$ d) $\frac{24}{9}=\frac{18}{9}+\frac{6}{9}=2\frac{6}{9}$

4) Marisela cut four equivalent lengths of ribbon. Each was 5 eighths of a yard long. How many yards of ribbon did she cut? Express your answer as the sum of a whole number and the remaining fractional units. Draw a number line to represent the problem.

$4\times\frac{5}{8}=\frac{20}{8}=\frac{16}{8}+\frac{4}{8}=2+\frac{4}{8}$

Marisela's ribbon was $2\frac{4}{8}$ yards long.